Bouncing Universe and phantom crossing in Modified Gravity and its reconstruction

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Abstract

In this paper we consider FRW cosmology in modified gravity which contain arbitrary functions $f(\phi)$. It is shown that the bouncing solution appears in the model whereas the equation of state (EoS) parameter crosses the phantom divider. The reconstruction of the model is also investigated with the aim to reconstruct the arbitrary functions and variables of the model.

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1. INTRODUCTION

There are many cosmological observations, such as Super-Nova Ia (SNIa) [1], Wilkinson Microwave Anisotropy Probe (WMAP) [2], Sloan Digital Sky Survey (SDSS) [3], Chandra X-ray Observatory [4] etc., that reveal some cross-checked information of our universe. They suggest that the universe is spatially flat, and consists of approximately 70% dark energy (DE) with negative pressure, 30% dust matter (cold dark matters plus baryons), and negligible radiation, and also the universe is undergoing an accelerated expansion.

Recent observations have determined basic cosmological parameters in high-precisions, but at the same time they posed a serious problem about the origin of DE. The combined analysis of SNIa [5], that is based upon the background expansion history of the universe around the redshift $z < \mathcal{O}(1)$, galaxy clusters measurements and WMAP data, provides an evidence for the accelerated cosmic expansion [6]. The cosmological acceleration strongly indicates that the present day universe is dominated by smoothly distributed slowly varying DE component. The constraint obtained from SNIa so far has a degeneracy in the EoS of DE [7]. To many people's frustration, the ΛCDM model with an EoS $\omega = -1$ has been continuously favored from observations. This degeneracy has been present even by adding other constraints coming from Cosmic Microwave Background (CMB) [8] and Baryon Acoustic Oscillations (BAO) [9]. The modern constraints on the EoS parameter are around the cosmological constant value, $\omega = -1 \pm 0.1$ [6]-[10] and a possibility that ω is varied in time is not excluded. From the theoretical point of view there are three essentially different cases: $\omega > -1$ (quintessence), $\omega = -1$ (cosmological constant) and $\omega < -1$ (phantom) ([11]-[14] and refs. therein).

The models of DE can be broadly classified into two classes [15, 16]. The first corresponds to introducing a specific matter that leads to an accelerated expansion. Most of scalar field models such as quintessence [17] and k-essence [18] belong to this class. The second class, that in this paper we consider, corresponds to the so-called modified gravity models such as $f(R)$ gravity [19], scalar-tensor theories [20] and brane-world models [21]. In order to break the degeneracy of observational constraints on ω and to discriminate between a DE models, it is important to find additional information other than the background expansion history of the Universe [22].

In second classification, modified gravity [23] suggests fine alternative for DE origin.

Indeed, it may be naturally expected that gravitational action contains some extra terms which became relevant recently with the significant decrease of the universe curvature. The modified gravity can be obtained in two ways, first by replacing scalar curvature R, or with $f(R)$, in the action which is well known as modified gravity ,or $f(R)$ modified gravity, and second by considering additional curvature invariant terms like Gauss-Bonnet (GB) term. Another modification of GR is modified Gauss-Bonnet gravity [24] which is obtained by inserting a function of GB invariant $f(G)$, in the Einstein-Hilbert action. A number of metric formulation of modified $f(R)$ gravities has been proposed [23]-[28] which explain the origin of cosmic acceleration. Particular attention is paid to $f(R)$ models [29]-[32] with the effective cosmological constant phase because such theories may easily reproduce the well-known ΛCDM cosmology. Such models subclass which does not violate Solar System tests represents the real alternative for standard General Relativity.[33]

On the other hand, the Friedman equation forms the starting point for almost all investigations in cosmology. Over the past few years possible corrections to the Friedman equation have been derived or proposed in a number of different contexts, generally inspired by brane-world investigation [34, 35]. These modification are often of a form that involves the total energy density ρ. In [36], multi-scalar coupled to gravity is studied in the context of conventional Friedman cosmology. It is found that the cosmological trajectories can be viewed as geodesic motion in an augmented target space.

There are several phenomenological models describing the crossing of the cosmological constant barrier [37, 38]. Most of them use more then one scalar field or use a non-minimal coupling with the gravity, or modified gravity, however we use both of them in our paper. In two-field models one of these two fields is a phantom, other one is a usual field and the interaction is non-polynomial in general. It is important to find a model which follows from the fundamental principles and describes a crossing of the $\omega = -1$ barrier.

A bouncing universe which provides a possible solution to the Big Bang singularity problem in standard cosmology has recently attracted a lot of interest in the field of string theory and modified gravity [39, 40]. In bouncing cosmology, within the framework of the standard FRW cosmology the null energy condition (NEC) for a period of time around the bouncing point is violated . Moreover, after the bouncing when the universe enters into the hot Big Bang era, the EoS parameter ω in the universe transits from $\omega < -1$ to $\omega > -1$ [41].

In this paper, in section 2 we study the dynamics of the FRW cosmology in modified

(non-local) gravity. We discuss analytically and numerically a detailed examination of the conditions for having ω across over -1. The necessary conditions required for a successful bounce is discussed in this section as well. Section 3 describes our model reconstruction and present the results. Finally, we summaries our paper in section 4.

2. THE MODEL

We start with the action of the non-local gravity as a simple modified garvity given by [33],

$$S = \int d^4x\sqrt{-g}\left\{\frac{M_p^2}{2}R(1+f(\Box^{-1}R))\right\}\cdot \tag{1}$$

where M_p is Plank mass, f is some function and $\Box$ is d'Almbertian for scalar field. Generally speaking, such non-local effective action, derived from string theory, may be induced by quantum effects. A Bi- scalar reformation of non-local action can be presented by introducing two scalar fields ϕ and ψ, where changes the above action to a local from:

$$S = \int d^4x\sqrt{-g}\left\{\frac{M_p^2}{2}\left[R(1+f(\phi))+\psi(\Box\phi-R)\right]\right\}, \tag{2}$$

where ψ, at this stage, plays role of a lagrange multiplier. One might further rewrite the above action as

$$S = \int d^4x\sqrt{-g}\left\{\frac{M_p^2}{2}\left[R(1+f(\phi)-\psi)-\partial_\mu\psi\partial^\mu\phi\right]\right\},\cdot \tag{3}$$

which now is equivalent to a local model with two extra degrees of freedom. By the variation over ψ, we obtain $\Box\phi = R$ or $\phi = \Box^{-1}R$, where f is a function of scalar field ϕ in the model.

Now in a FRW cosmological model with invariance of the action under changing fields and vanishing variations at the boundary, the equations of motion for only time dependent scalar fields, ϕ and ψ, become

$$\ddot{\phi}+3H\dot{\phi}+R = 0, \tag{4}$$

$$\ddot{\psi}+3H\dot{\psi}-Rf' = 0, \tag{5}$$

where $R=12H^2+6\dot{H}$, H is Hubble parameter and $f'=\frac{df(\phi)}{d\phi}$. Variation of action (3) with respect to the metric tensor $g_{\mu\nu}$ gives,

$$\begin{aligned}0 &= \frac{1}{2}g_{\mu\nu}\left\{R(1+f-\psi)-\partial_\rho\psi\partial^\rho\phi\right\}-R_{\mu\nu}(1+f-\psi)+\frac{1}{2}(\partial_\mu\psi\partial_\nu\phi+\partial_\mu\phi\partial_\nu\psi)\\ &- (g_{\mu\nu}\Box-\nabla_\mu\nabla_\nu)(f-\psi)\cdot\end{aligned} \tag{6}$$

The 00 and ii components of the equation (6) are

$$0 = -3H^2(1+f-\psi) + \frac{1}{2}\dot{\psi}\dot{\phi} - 3H(f'\dot{\phi} - \dot{\psi}), \tag{7}$$

$$0 = (2\dot{H} + 3H^2)(1+f-\psi) + \frac{1}{2}\dot{\psi}\dot{\phi} + (\frac{d^2}{dt^2} + 2H\frac{d}{dt})(f-\psi)\cdot \tag{8}$$

Equations (7) and (8) can be rewritten as

$$3H^2 = \frac{\frac{1}{2}\dot{\psi}\dot{\phi} - 3H(f'\dot{\phi} - \dot{\psi})}{(1+f-\psi)}, \tag{9}$$

$$2\dot{H} + 3H^2 = -\frac{\frac{1}{2}\dot{\psi}\dot{\phi} + (\frac{d^2}{dt^2} + 2H\frac{d}{dt})(f-\psi)}{(1+f-\psi)}\cdot \tag{10}$$

Comparison with the standard Friedman equations $H^2 = \frac{\rho_{eff}}{3M_p^2}$, and $2\dot{H} + 3H^2 = -\frac{p_{eff}}{2M_p^2}$, the right hand side of the equations (9) and (10) can be treated as the effective energy density and pressure:

$$\frac{\rho_{eff}}{M_p^2} = \frac{\frac{1}{2}\dot{\psi}\dot{\phi} - 3H(f'\dot{\phi} - \dot{\psi})}{(1+f-\psi)}, \tag{11}$$

$$\frac{p_{eff}}{M_p^2} = \frac{\frac{1}{2}\dot{\psi}\dot{\phi} + (\frac{d^2}{dt^2} + 2H\frac{d}{dt})(f-\psi)}{(1+f-\psi)}\cdot \tag{12}$$

Using Eqs. (4) and (5) and doing some algebraic calculation we can read the effective energy density and pressure from the above as,

$$\rho_{eff} = \frac{M_p^2}{1+f-\psi}\left\{\frac{1}{2}\dot{\psi}\dot{\phi} - 3H(f'\dot{\phi} - \dot{\psi})\right\}\cdot \tag{13}$$

$$p_{eff} = \frac{M_p^2}{1+f-\psi-6f'}\left\{\frac{1}{2}\dot{\psi}\dot{\phi} + f''\dot{\phi}^2 - H(f'\dot{\phi} - \dot{\psi}) + \frac{f'\left[6H(f'\dot{\phi} - \dot{\psi}) - \dot{\psi}\dot{\phi}\right]}{1+f-\psi}\right\}\cdot \tag{14}$$

Now by using Eqs. (13) and (14) the conservation equation can be obtained as,

$$\dot{\rho}_{eff} + 3H\rho_{eff}(1+\omega) = 0, \tag{15}$$

where $\omega = \frac{p_{eff}}{\rho_{eff}}$ is the EoS parameter of the model. Also from Eq. (7) we obtain,

$$H = \frac{-(f'\dot{\phi} - \dot{\psi})}{2(1+f-\psi)}\left\{1 \pm \sqrt{1 + \frac{6}{9}\frac{\dot{\psi}\dot{\phi}(1+f-\psi)}{(f'\dot{\phi} - \dot{\psi})^2}}\right\}\cdot \tag{16}$$

At this stage we study the cosmological evolution of EoS parameter, ω , and show that analytically and numerically there are conditions that cause the EoS parameter crosses the

phantom divide line ($\omega \to -1$). Let's see under what conditions the system will be able to cross the barrier of $\omega = -1$. In order to do that, one requires $\rho_{eff} + p_{eff}$ to vanish at a point of (ϕ_0, ψ_0) and change the sign after the crossing. This can only be achieved by requiring $\dot{H}(\phi_0, \psi_0) = 0$ and $\dot{H}$ has different signs before and after the crossing.

To explore this possibility, we have to check the condition $\frac{d}{dt}(\rho_{eff} + p_{eff}) \neq 0$ when $\omega \to -1$. Using Eqs. (13) and (14) in second Friedman equation, Eq. (10) gives,

$$\dot{H} = \frac{24f'H^2 + 4H(f'\dot{\phi} - \dot{\psi}) - (f''\dot{\phi}^2 + \dot{\psi}\dot{\phi})}{2(1 + f - \psi - 6f')}. \tag{17}$$

Also we have $\frac{d}{dt}(\rho_{eff} + p_{eff}) = -2M_p^2\ddot{H}$ or,

$$\begin{aligned}\ddot{H} &= \frac{\ddot{\phi}(4Hf' - 2f''\dot{\phi} - \dot{\psi}) - \ddot{\psi}(4H + \dot{\phi}) + \dot{\phi}\Big(4Hf''(6H + \dot{\phi}) - f'''\dot{\phi}^2\Big)}{2(1 + f - \psi - 6f')} \\ &+ \dot{H}\left(\frac{24f'H + \dot{\phi}(f' + 6f'') - \dot{\psi}}{1 + f - \psi - 6f'}\right). \end{aligned}\tag{18}$$

One can find that in order to have ω-crossing one of the following conditions might be satisfied when $\omega \to -1$ and $H \neq 0$.

- (a) $\dot{\psi} = 0$ and $\dot{\phi} \neq 0$
- (b) $\dot{\phi} = 0$ and $\dot{\psi} \neq 0$
- (c) $\ddot{\phi} = 0$ and $\ddot{\psi} = 0$

In the first case, (a),we have

$$H = \frac{-f'\dot{\phi}}{1 + f - \psi}, \tag{19}$$

$$\dot{H} = \frac{4H(6f'H + \dot{\phi}) - f''\dot{\phi}^2}{2(1 + f - \psi - 6f')}, \tag{20}$$

and,

$$\ddot{H} = \frac{2\ddot{\phi}(2Hf' - f''\dot{\phi}) - \ddot{\psi}(4H + \dot{\phi}) + \dot{\phi}\left(4Hf''(6H + \dot{\phi}) - f'''\dot{\phi}^2\right)}{2(1 + f - \psi - 6f')}. \tag{21}$$

Therefore the conditions for having ω across over -1 are: (a-1) $\ddot{\phi} \neq 0$ and $2Hf' \neq f''\dot{\phi}$ when other terms can be neglected, (a-2) $\ddot{\psi} \neq 0$ however the first and third terms can be

vanished, (a-3) $2\ddot{\phi}(2Hf'-f''\dot{\phi}) \neq \ddot{\psi}(4H+\dot{\phi})$ where third term is zero or (a-4) $4Hf''(6H+\dot{\phi}) \neq f'''\dot{\phi}^2$ or if $f'''=0$ then $f'' \neq 0$ and if $f''=0$ then $f''' \neq 0$, when the first and second terms can be vanished in addition to the $f''\dot{\phi}^2 = 4H(6Hf' + \dot{\phi})$ and $1+f-\psi \neq 6f'$.

In the second case, (b), we have,

$$H = \frac{\dot{\psi}}{1+f-\psi}, \tag{22}$$

$$\dot{H} = \frac{2H(6f'H - \dot{\psi})}{1+f-\psi-6f'}, \tag{23}$$

and,

$$\ddot{H} \;=\; \frac{\ddot{\phi}(4Hf' - \dot{\psi}) - 4H\ddot{\psi}}{2(1+f-\psi-6f')}. \tag{24}$$

This case never occurs because when $\dot{H} = 0$ and $H \neq 0$ we obtain $\dot{\psi} = 6f'H$. By replacing it in Eq. (22) one leads to $6f' = 1+f-\psi$ which contradict our primary finding.

Finally, for the third case, (c), we have, H and $\dot{H}$ as Eqs. (16) and (17) in addition to,

$$\ddot{H} = \frac{\dot{\phi}\left(4Hf''(6H+\dot{\phi}) - f'''\dot{\phi}^2\right)}{2(1+f-\psi-6f')}. \tag{25}$$

Therefore the conditions are (c-1) $4Hf''(6H+\dot{\phi}) \neq f'''\dot{\phi}^2$, (c-2) $f'''=0$ and $f'' \neq 0$, (c-3) $f''=0$ and $f''' \neq 0$, (c-4) $\dot{\phi} \neq 0$, in addition to the $4Hf'(6H^2+\dot{\phi}) - f''\dot{\phi}^2 = \dot{\psi}(4H+\dot{\phi})$ and $1+f-\psi \neq 6f'$.

With numerical calculation, as shown in Fig. 1, by appropriately choosing model parameters we construct a cosmological model so that crossing the phantom divide occurs at $t > 0$ and crosses -1 around this point which is supported by observations [14]. In some sense this model is similar to Quintom dark energy models consisting of two quintessence and phantom fields [42].

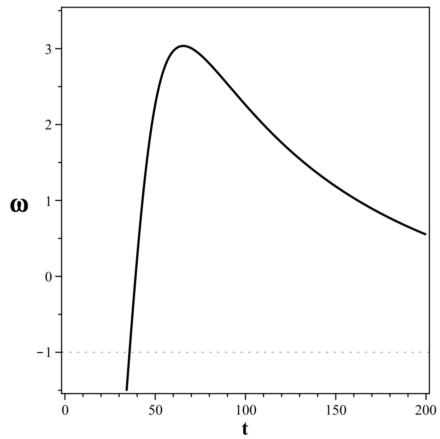

Fig. 1: The graph of ω plotted as function of time for $f_0\, \mathrm{e}^{b\phi(t)}$, $f_0 = 0.5$ and $b = 1.5$. Initial values are $\phi(0) = 0.5$, $\dot{\phi}(0) = -0.02$, $\psi(0) = 0.5$, $\dot{\psi}(0) = 0.01$.

By choosing $t = 0$ to be the bouncing point, the solution for $a(t)$ and $H(t)$, Eq. (16), (see Fig. 2) provides a dynamical universe with contraction for $t < 0$, bouncing at $t = 0$ and then expansion for $t > 0$.

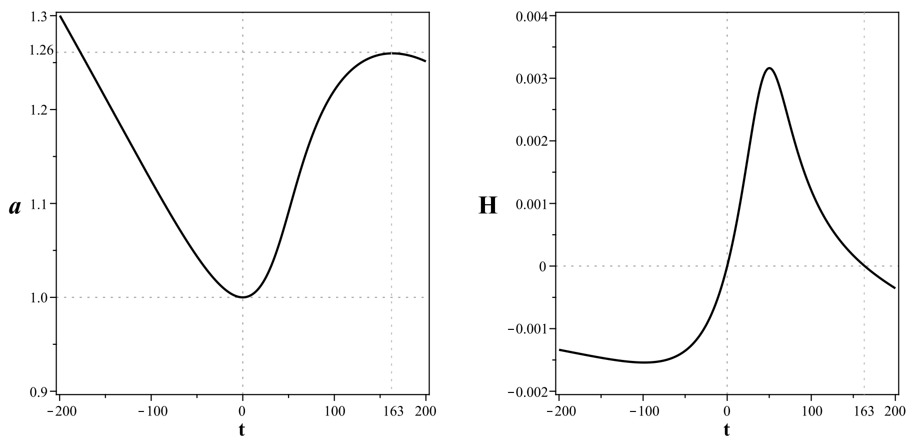

Fig. 2: The graph of scale factor a and H, plotted as function of time for $f_0\, \mathrm{e}^{b\phi(t)}$, $f_0 = 0.5$, and $b = 1.5$. Initial values are $\phi(0) = 0.5$, $\dot{\phi}(0) = -0.02$, $\psi(0) = 0.5$, $\dot{\psi}(0) = 0.01$.

By definition, for a bounce or turn-around process to occur, one must require that at the pivot point $\dot{a} = 0$ and $\ddot{a} > 0$ around the bouncing point, while $\ddot{a} < 0$ around the turn-around point. According to Eqs. (9) and (10), for $\frac{\dot{\psi}\dot{\phi}}{2(1+f-\psi)} > 0$ one can get

$$\rho_{eff} > 0, \quad p_{eff} < 0 \quad (\text{ or } \quad p_{eff} > 0) \quad \text{for the bounce (or turn-around)} \tag{26}$$

or equivalently, the EoS parameter approaches negative values (or positive values) at the bounce (or turn-around) point. This shows ω possibly crosses over the cosmological constant

boundary ($\omega = -1$), which interestingly implies the necessity to have the Quintom matter for the realization of the oscillating universe under modified gravity.

In the context of a spatially flat four-dimensional background FRW metric, if we are to obtain a smooth transition from a contracting universe into an expanding phase, there must be a period when NEC is violated . In this case, we need a kind of matter which admits an EoS parameter which is less than -1, but only around the bounce. Neither regular not Phantom matter alone can achieve a transition in the EoS parameter through the cosmological constant boundary. Therefore, a Quintom model is the only possible solution to resolve this difficulty.

A detailed examination on the necessary conditions requires for a successful bounce shows that during the contracting phase, the scale factor $a(t)$ is decreasing, i.e., $\dot{a} < 0$, and in the expanding phase we have $\dot{a} > 0$. At the bouncing point, $\dot{a} = 0$, and so around this point $\ddot{a} > 0$ for a period of time. Equivalently in the bouncing cosmology the Hubble parameter H runs across zero from $H < 0$ to $H > 0$ and $H = 0$ at the bouncing point. A successful bounce requires that the following condition should be satisfied around bouncing point,

$$\dot{H} = -\frac{1}{2M_p^2}(1+\omega)\rho > 0. \tag{27}$$

From Fig. 1 and 2, we see that at $t \to 0$, $\omega < -1$ and $\dot{H}$ is positive which satisfies the above condition. Also we see that at the bouncing point where the scale factor $a(t)$ is not zero we avoid singularity faced in the standard cosmology.

3. MODEL RECONSTRUCTION

Reconstructing of the model for the EoS parameter and deceleration parameter in three forms of parametrization [43] has been studied in this section. From effective energy density and effective pressure, Eqs, (13) and (14) one can define construction function $\tilde{K}$ as:

$$3p_{eff} - \rho_{eff} = \frac{M_p^2}{1+f-\psi}(\dot{\psi}\dot{\phi} + 6(\ddot{f} + 3H\dot{f})) = 2\tilde{K}\cdot \tag{28}$$

With comparison to Eq. (13) and using Eq. (16) we have,

$$\rho_{eff} = \tilde{K} - 3\tilde{V}, \tag{29}$$

where,

$$\tilde{V} = \frac{M_p^2}{1+f-\psi}\left(\ddot{f} + 4H\dot{f} - H\dot{\psi}\right)\cdot \tag{30}$$

One then simply finds that the energy pressure in terms of new functions $\tilde{K}$ an $\tilde{V}$ as,

$$p_{eff} = \tilde{K} - \tilde{V}, \tag{31}$$

now we can rewrite the EoS parameter as,

$$\omega = -1 + \left(\frac{2\tilde{K} - 4\tilde{V}}{\tilde{K} - 3\tilde{V}} \right) \cdot \tag{32}$$

It can be seen that $\omega > -1$ when $\tilde{K} > 3\tilde{V}$ or $\tilde{K} < 2\tilde{V}$ and $\omega < -1$ when $2\tilde{V} < \tilde{K} < 3\tilde{V}$ with the constrain that $\tilde{K} \neq 3\tilde{V}$. The transition from $\omega > -1$ to $\omega < -1$ happens when $\tilde{K} = 2\tilde{V}$.

In order to reconstruct the cosmological parameters we rewrite the modified Friedman equations in the present of dust matter as,

$$\begin{aligned} 3M_p^2 H^2 &= \rho_m + \rho_{eff} = \rho_m + \tilde{K} - 3\tilde{V}, & (33) \\ -2M_p^2 \dot{H} &= \rho_m + \rho_{eff} + p_{eff} = \rho_m + 2\tilde{K} - 4\tilde{V}, & (34) \end{aligned}$$

where ρ_m is the energy density of dust matter. We thus have new $\tilde{K}$ and $\tilde{V}$ in the present of matter:

$$\begin{aligned} \tilde{K} &= \frac{1}{2}\rho_m - 3M_p^2(2H^2 + \dot{H}), & (35) \\ \tilde{V} &= \frac{1}{2}\rho_m - M_p^2(3H^2 + \dot{H}) \cdot & (36) \end{aligned}$$

In here, we assume that the two effective and dust matter fluids do not interact. From [44] the expression for the energy density of dust matter with respect to the redshift z is given by,

$$\rho_m = 3M_p^2 H_0^2 \Omega_{m0}(1+z)^3, \tag{37}$$

where Ω_{m0} is the ratio density parameter of matter fluid and the subscript 0 indicates the present value of the corresponding quantity. One then can rewrite $\tilde{K}$ and $\tilde{V}$ with respect to the redshift z as,

$$\begin{aligned} \tilde{K} &= \frac{3}{2}M_p^2 H_0^2 \left(\Omega_{m0}(1+z)^3 - 4r + (1+z)r^{(1)} \right), & (38) \\ \tilde{V} &= \frac{1}{2}M_p^2 H_0^2 \left(3\Omega_{m0}(1+z)^3 - 6r + (1+z)r^{(1)} \right), & (39) \end{aligned}$$

where $r = \frac{H^2}{H_0^2}$ and $r^{(n)} = \frac{d^n r}{dz^n}$. By using Eqs. (38) and (39) the EoS parameter can be rewritten as,

$$\omega = \frac{(1+z)r^{(1)} - 3r}{-3\Omega_{m0}(1+z)^3 + 3r}, \tag{40}$$

where then $r(z)$ can be evaluated

$$r(z) = \Omega_{m0}(1+z)^3 + (1-\Omega_{m0})e^{3\int_0^z \frac{1+\omega(\tilde{z})}{1+\tilde{z}}d\tilde{z}}. \tag{41}$$

Moreover, by employing $\frac{d}{dt} = -(1+z)\frac{d}{dz}$, the deceleration parameter q can be obtained as,

$$q(z) = -1 - \frac{\dot{H}}{H^2} = \frac{(1+z)r^{(1)} - 2r}{2r}\cdot \tag{42}$$

Now, with the following three different forms of parametrization, using numerical calculation, we reconstruct EoS and $q(z)$ parameters.

Parametrization 1:

This parametrization has been proposed by Chevallier and Polarski [45] and Linder [46], where the EoS parameter of dark energy in term of redshift z is given by,

$$\omega(z) = \omega_0 + \frac{\omega_a z}{1+z}\cdot \tag{43}$$

By fitting this model to the observational data we find that $\Omega_{m0} = 0.29$, $\omega_0 = -1.07$ and $\omega_a = 0.85$ [47]

Parametrization 2:

The EoS parameter in term of redshift z has been proposed by Jassal, Bagla and Padmanabhan [48] as,

$$\omega(z) = \omega_0 + \frac{\omega_b z}{(1+z)^2}\cdot \tag{44}$$

where again fitting the data [47], $\Omega_{m0} = 0.28$, $\omega_0 = -1.37$ and $\omega_b = 3.39$

Parametrization 3:

The third parametrization has proposed by Alam, Sahni and Starobinsky [49], where $r(z)$ given by,

$$r(z) = \Omega_{m0}(1+z)^3 + A_0 + A_1(1+z) + A_2(1+z)^2\cdot \tag{45}$$

This parametrization can be thought as the parametrization of $r(z)$ instead of $\omega(z)$. By using the results in [47], we get coefficients of the this parametrization as $\Omega_{m0} = 0.30$, $A_0 = 1$, $A_1 = -0.48$ and $A_2 = 0.25$.

The evolution of $\omega(z)$ and $q(z)$ are plotted in Fig. 3 for the three form of parameterizations. The figure shows that the EoS parameter crosses the phantom divide line for the first and second parametrization and never crosses the line in the third parametrization. The second parametrization crosses the phantom line in two different values of z. As can be seen from the graph of deceleration parameter, for the second parametrization the universe undergoes an acceleration period from $z = 0.39$ until now in comparison to the first and third parameterizations that acceleration starts earlier.

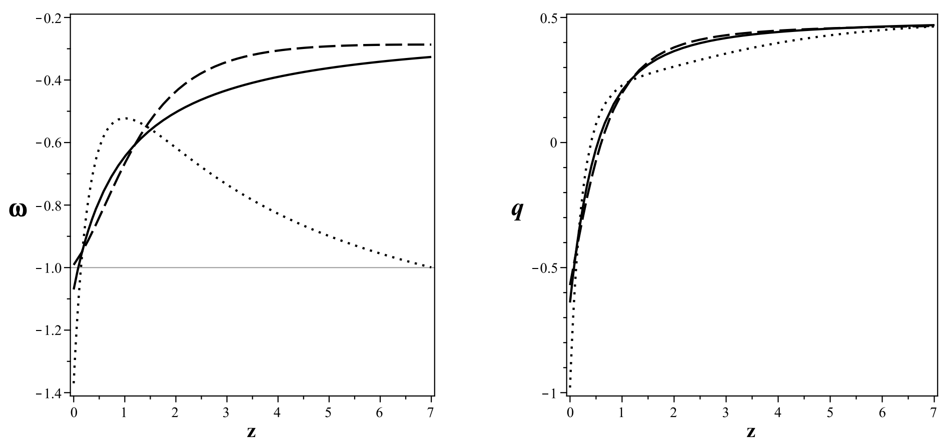

Fig. 3: The graphs of the EoS parameters, ω, and deceleration parameters, q, with respect to the redshift z. The solid, dot and dash lines represent parametrization 1, 2 and 3 respectively.

Also, using Eqs. (38), (39) and the three parameterizations, the evolutions of $K(\tilde{z})$ and $V(\tilde{z})$ are shown in Fig. 4.

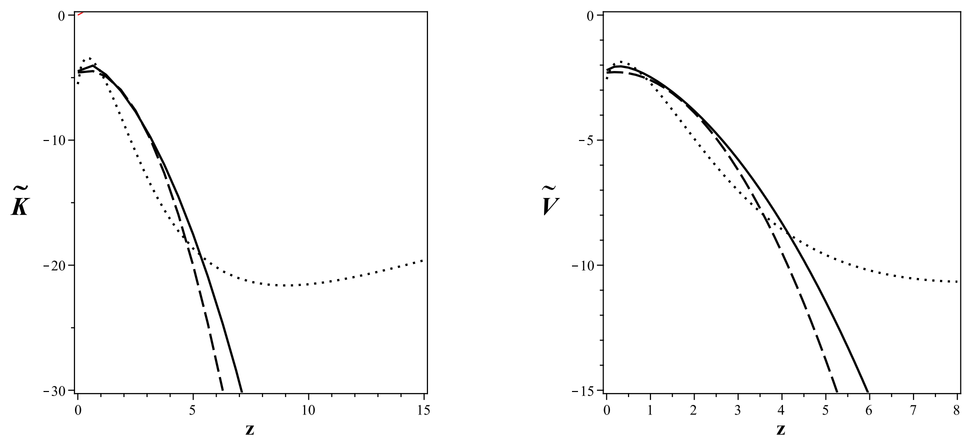

Fig. 4: The graphs of the reconstructed $\tilde{K}$ and $\tilde{V}$ with respect to the redshift z. The solid, dot and dash lines represent parametrization 1, 2 and 3 respectively.

Since $\tilde{K}$ and $\tilde{V}$ are now known functions of z, we can obtain the evolutions of $\phi(z)$ and $f(z)$ with respect of z, which are plotted in Fig. 5 for the three parameterizations. One may also directly obtain the relationship between the function f and the scalar field ϕ, which is plotted in Fig. 6 for the three parameterizations.

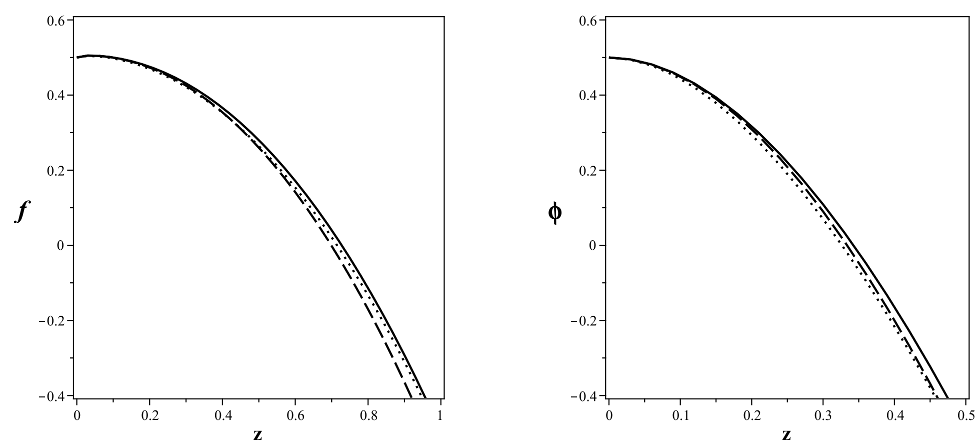

Fig. 5: Graphs for the reconstructed f and ϕ in respect of z. The solid, dot and dash lines represent parametrization 1, 2 and 3 respectively. Initial values are $\phi(0) = 0.5$, $\dot{\phi}(0) = -0.02$, $\psi(0) = 0.5$, $\dot{\psi}(0) = 0.01$ and $f(0) = 0.5$.

One then can reconstruct $f(\phi)$, which is plotted in Fig. 6. The exponential behavior of the reconstructed $f(\phi)$with respected to ϕ in particular for the second parametrization is compatible with the one initially assumed in the numerical calculations.

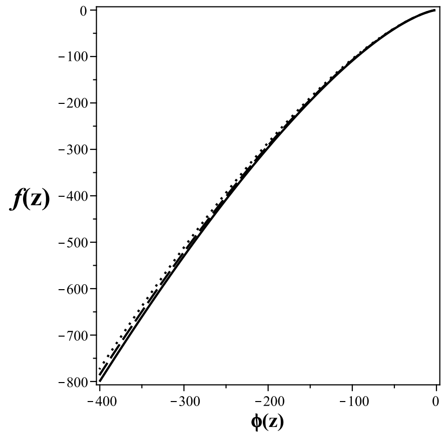

Fig. 6: Graphs for the reconstructed f in respect of ϕ. The solid, dot and dash lines represent parametrization 1, 2 and 3 respectively. Initial values are $\phi(0) = 0.5$, $\dot{\phi}(0) = -0.02$, $\psi(0) = 0.5$, $\dot{\psi}(0) = 0.01$ and $f(0) = 0.5$.

4. SUMMARY AND CONCLUSION

In this paper , we consider a local scalar-tensor formulation of non-local gravity as a simple modified model characterized by two scalar fields ϕ and ψ and function $f(\phi)$ which can be viewed as scalar potential in the model. Analytical study of the solution shows that under special condition, the universe may go through a transition from quintessence to phantom phase which is also supported by numerical analysis.
In analytic studying of the dynamics of the EoS parameter we achieve the constraints that one has to impose on the scalar fields and their first and second derivatives in order to have phantom crossing. In numerical approach, the EoS parameter crosses $\omega = -1$ for $t > 0$. We also find that universe undergoes a bounce, i.e contracts, reaches a minimum radius and then expands.

Finally, we reconstruct our cosmological parameters, such as the EoS parameter, deceleration parameter and potential function $f(\phi)$. In general, different cosmological models can be observationally differentiate in terms of the potential function of the dynamical system. In here, in the construction of the deceleration parameter $q(z)$, it is found that the strongest evidence of acceleration occurs about redshift $z \sim 0.2-0.39$ [47]. Also for the EoS parameter, it is found that $\omega(z)$ crossing to be around redshift $z \sim 0.2$ [47]. This suggest that all three parametrization are suitable for small z and in particular the second one is

remarkably close to the observational data. We then reconstructed the potential function of the dynamical system using three forms of parametrization by fitting the model to the observational. The behavior of $f(\phi)$ is similar to the initially assumed exponential form.

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